

AP Calculus AB/BC Review

Rate and Quantity

SOLUTIONS AND
SCORING

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2015 SCORING GUIDELINES

Question 3

t (minutes)	0	12	20	24	40
$v(t)$ (meters per minute)	0	200	240	-220	150

Johanna jogs along a straight path. For $0 \leq t \leq 40$, Johanna's velocity is given by a differentiable function v . Selected values of $v(t)$, where t is measured in minutes and $v(t)$ is measured in meters per minute, are given in the table above.

- (a) Use the data in the table to estimate the value of $v'(16)$.
- (b) Using correct units, explain the meaning of the definite integral $\int_0^{40} |v(t)| dt$ in the context of the problem.

Approximate the value of $\int_0^{40} |v(t)| dt$ using a right Riemann sum with the four subintervals indicated in the table.

- (c) Bob is riding his bicycle along the same path. For $0 \leq t \leq 10$, Bob's velocity is modeled by $B(t) = t^3 - 6t^2 + 300$, where t is measured in minutes and $B(t)$ is measured in meters per minute.

Find Bob's acceleration at time $t = 5$.

- (d) Based on the model B from part (c), find Bob's average velocity during the interval $0 \leq t \leq 10$.

(a) $v'(16) \approx \frac{240 - 200}{20 - 12} = 5 \text{ meters/min}^2$

1 : approximation

- (b) $\int_0^{40} |v(t)| dt$ is the total distance Johanna jogs, in meters, over the time interval $0 \leq t \leq 40$ minutes.

3 : $\begin{cases} 1 : \text{explanation} \\ 1 : \text{right Riemann sum} \\ 1 : \text{approximation} \end{cases}$

$$\begin{aligned} \int_0^{40} |v(t)| dt &\approx 12 \cdot |v(12)| + 8 \cdot |v(20)| + 4 \cdot |v(24)| + 16 \cdot |v(40)| \\ &= 12 \cdot 200 + 8 \cdot 240 + 4 \cdot 220 + 16 \cdot 150 \\ &= 2400 + 1920 + 880 + 2400 \\ &= 7600 \text{ meters} \end{aligned}$$

- (c) Bob's acceleration is $B'(t) = 3t^2 - 12t$.
 $B'(5) = 3(25) - 12(5) = 15 \text{ meters/min}^2$

2 : $\begin{cases} 1 : \text{uses } B'(t) \\ 1 : \text{answer} \end{cases}$

(d) Avg vel $= \frac{1}{10} \int_0^{10} (t^3 - 6t^2 + 300) dt$

$$\begin{aligned} &= \frac{1}{10} \left[\frac{t^4}{4} - 2t^3 + 300t \right]_0^{10} \\ &= \frac{1}{10} \left[\frac{10000}{4} - 2000 + 3000 \right] = 350 \text{ meters/min} \end{aligned}$$

3 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{antiderivative} \\ 1 : \text{answer} \end{cases}$

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Question 1

The rate at which rainwater flows into a drainpipe is modeled by the function R , where $R(t) = 20\sin\left(\frac{t^2}{35}\right)$ cubic feet per hour, t is measured in hours, and $0 \leq t \leq 8$. The pipe is partially blocked, allowing water to drain out the other end of the pipe at a rate modeled by $D(t) = -0.04t^3 + 0.4t^2 + 0.96t$ cubic feet per hour, for $0 \leq t \leq 8$. There are 30 cubic feet of water in the pipe at time $t = 0$.

- (a) How many cubic feet of rainwater flow into the pipe during the 8-hour time interval $0 \leq t \leq 8$?
- (b) Is the amount of water in the pipe increasing or decreasing at time $t = 3$ hours? Give a reason for your answer.
- (c) At what time t , $0 \leq t \leq 8$, is the amount of water in the pipe at a minimum? Justify your answer.
- (d) The pipe can hold 50 cubic feet of water before overflowing. For $t > 8$, water continues to flow into and out of the pipe at the given rates until the pipe begins to overflow. Write, but do not solve, an equation involving one or more integrals that gives the time w when the pipe will begin to overflow.

(a) $\int_0^8 R(t) dt = 76.570$

2 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $R(3) - D(3) = -0.313632 < 0$
 Since $R(3) < D(3)$, the amount of water in the pipe is decreasing at time $t = 3$ hours.

2 : $\begin{cases} 1 : \text{considers } R(3) \text{ and } D(3) \\ 1 : \text{answer and reason} \end{cases}$

(c) The amount of water in the pipe at time t , $0 \leq t \leq 8$, is
 $30 + \int_0^t [R(x) - D(x)] dx$.

3 : $\begin{cases} 1 : \text{considers } R(t) - D(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

$R(t) - D(t) = 0 \Rightarrow t = 0, 3.271658$

t	Amount of water in the pipe
0	30
3.271658	27.964561
8	48.543686

The amount of water in the pipe is a minimum at time $t = 3.272$ (or 3.271) hours.

(d) $30 + \int_0^w [R(t) - D(t)] dt = 50$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{equation} \end{cases}$

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Question 4

Train A runs back and forth on an east-west section of railroad track. Train A 's velocity, measured in meters per minute, is given by a differentiable function $v_A(t)$, where time t is measured in minutes. Selected values for $v_A(t)$ are given in the table above.

t (minutes)	0	2	5	8	12
$v_A(t)$ (meters/minute)	0	100	40	-120	-150

- (a) Find the average acceleration of train A over the interval $2 \leq t \leq 8$.
- (b) Do the data in the table support the conclusion that train A 's velocity is -100 meters per minute at some time t with $5 < t < 8$? Give a reason for your answer.
- (c) At time $t = 2$, train A 's position is 300 meters east of the Origin Station, and the train is moving to the east. Write an expression involving an integral that gives the position of train A , in meters from the Origin Station, at time $t = 12$. Use a trapezoidal sum with three subintervals indicated by the table to approximate the position of the train at time $t = 12$.
- (d) A second train, train B , travels north from the Origin Station. At time t the velocity of train B is given by $v_B(t) = -5t^2 + 60t + 25$, and at time $t = 2$ the train is 400 meters north of the station. Find the rate, in meters per minute, at which the distance between train A and train B is changing at time $t = 2$.

(a) average accel = $\frac{v_A(8) - v_A(2)}{8 - 2} = \frac{-120 - 100}{6} = -\frac{110}{3} \text{ m/min}^2$

1 : average acceleration

(b) v_A is differentiable $\Rightarrow v_A$ is continuous
 $v_A(8) = -120 < -100 < 40 = v_A(5)$

2 : $\begin{cases} 1 : v_A(8) < -100 < v_A(5) \\ 1 : \text{conclusion, using IVT} \end{cases}$

Therefore, by the Intermediate Value Theorem, there is a time t , $5 < t < 8$, such that $v_A(t) = -100$.

(c) $s_A(12) = s_A(2) + \int_2^{12} v_A(t) dt = 300 + \int_2^{12} v_A(t) dt$
 $\int_2^{12} v_A(t) dt \approx 3 \cdot \frac{100 + 40}{2} + 3 \cdot \frac{40 - 120}{2} + 4 \cdot \frac{-120 - 150}{2}$
 $= -450$

3 : $\begin{cases} 1 : \text{position expression} \\ 1 : \text{trapezoidal sum} \\ 1 : \text{position at time } t = 12 \end{cases}$

$s_A(12) \approx 300 - 450 = -150$

The position of Train A at time $t = 12$ minutes is approximately 150 meters west of Origin Station.

- (d) Let x be train A 's position, y train B 's position, and z the distance between train A and train B .

$z^2 = x^2 + y^2 \Rightarrow 2z \frac{dz}{dt} = 2x \frac{dx}{dt} + 2y \frac{dy}{dt}$

$x = 300, y = 400 \Rightarrow z = 500$

$v_B(2) = -20 + 120 + 25 = 125$

$500 \frac{dz}{dt} = (300)(100) + (400)(125)$

$\frac{dz}{dt} = \frac{80000}{500} = 160 \text{ meters per minute}$

3 : $\begin{cases} 2 : \text{implicit differentiation of} \\ \text{distance relationship} \\ 1 : \text{answer} \end{cases}$

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2013 SCORING GUIDELINES

Question 3

t (minutes)	0	1	2	3	4	5	6
$C(t)$ (ounces)	0	5.3	8.8	11.2	12.8	13.8	14.5

Hot water is dripping through a coffeemaker, filling a large cup with coffee. The amount of coffee in the cup at time t , $0 \leq t \leq 6$, is given by a differentiable function C , where t is measured in minutes. Selected values of $C(t)$, measured in ounces, are given in the table above.

- (a) Use the data in the table to approximate $C'(3.5)$. Show the computations that lead to your answer, and indicate units of measure.
- (b) Is there a time t , $2 \leq t \leq 4$, at which $C'(t) = 2$? Justify your answer.
- (c) Use a midpoint sum with three subintervals of equal length indicated by the data in the table to approximate the value of $\frac{1}{6} \int_0^6 C(t) dt$. Using correct units, explain the meaning of $\frac{1}{6} \int_0^6 C(t) dt$ in the context of the problem.
- (d) The amount of coffee in the cup, in ounces, is modeled by $B(t) = 16 - 16e^{-0.4t}$. Using this model, find the rate at which the amount of coffee in the cup is changing when $t = 5$.

(a) $C'(3.5) \approx \frac{C(4) - C(3)}{4 - 3} = \frac{12.8 - 11.2}{1} = 1.6$ ounces/min

2 : $\begin{cases} 1 : \text{approximation} \\ 1 : \text{units} \end{cases}$

- (b) C is differentiable $\Rightarrow C$ is continuous (on the closed interval)

$$\frac{C(4) - C(2)}{4 - 2} = \frac{12.8 - 8.8}{2} = 2$$

Therefore, by the Mean Value Theorem, there is at least one time t , $2 < t < 4$, for which $C'(t) = 2$.

2 : $\begin{cases} 1 : \frac{C(4) - C(2)}{4 - 2} \\ 1 : \text{conclusion, using MVT} \end{cases}$

(c) $\frac{1}{6} \int_0^6 C(t) dt \approx \frac{1}{6} [2 \cdot C(1) + 2 \cdot C(3) + 2 \cdot C(5)]$

$$= \frac{1}{6} (2 \cdot 5.3 + 2 \cdot 11.2 + 2 \cdot 13.8)$$

$$= \frac{1}{6} (60.6) = 10.1 \text{ ounces}$$

$\frac{1}{6} \int_0^6 C(t) dt$ is the average amount of coffee in the cup, in ounces, over the time interval $0 \leq t \leq 6$ minutes.

3 : $\begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{approximation} \\ 1 : \text{interpretation} \end{cases}$

(d) $B'(t) = -16(-0.4)e^{-0.4t} = 6.4e^{-0.4t}$

$$B'(5) = 6.4e^{-0.4(5)} = \frac{6.4}{e^2} \text{ ounces/min}$$

2 : $\begin{cases} 1 : B'(t) \\ 1 : B'(5) \end{cases}$

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2012 SCORING GUIDELINES

Question 1

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W , where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F . The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

- (a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.
- (b) Use the data in the table to evaluate $\int_0^{20} W'(t) dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t) dt$ in the context of this problem.
- (c) For $0 \leq t \leq 20$, the average temperature of the water in the tub is $\frac{1}{20} \int_0^{20} W(t) dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20} \int_0^{20} W(t) dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.
- (d) For $20 \leq t \leq 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t} \cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

(a) $W'(12) \approx \frac{W(15) - W(9)}{15 - 9} = \frac{67.9 - 61.8}{6}$
 $= 1.017$ (or 1.016)

The water temperature is increasing at a rate of approximately 1.017°F per minute at time $t = 12$ minutes.

(b) $\int_0^{20} W'(t) dt = W(20) - W(0) = 71.0 - 55.0 = 16$

The water has warmed by 16°F over the interval from $t = 0$ to $t = 20$ minutes.

(c) $\frac{1}{20} \int_0^{20} W(t) dt \approx \frac{1}{20} (4 \cdot W(0) + 5 \cdot W(4) + 6 \cdot W(9) + 5 \cdot W(15))$
 $= \frac{1}{20} (4 \cdot 55.0 + 5 \cdot 57.1 + 6 \cdot 61.8 + 5 \cdot 67.9)$
 $= \frac{1}{20} \cdot 1215.8 = 60.79$

This approximation is an underestimate, because a left Riemann sum is used and the function W is strictly increasing.

(d) $W(25) = 71.0 + \int_{20}^{25} W'(t) dt$
 $= 71.0 + 2.043155 = 73.043$

2 : $\begin{cases} 1 : \text{estimate} \\ 1 : \text{interpretation with units} \end{cases}$

2 : $\begin{cases} 1 : \text{value} \\ 1 : \text{interpretation with units} \end{cases}$

3 : $\begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \\ 1 : \text{underestimate with reason} \end{cases}$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

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2011 SCORING GUIDELINES

Question 2

t (minutes)	0	2	5	9	10
$H(t)$ (degrees Celsius)	66	60	52	44	43

As a pot of tea cools, the temperature of the tea is modeled by a differentiable function H for $0 \leq t \leq 10$, where time t is measured in minutes and temperature $H(t)$ is measured in degrees Celsius. Values of $H(t)$ at selected values of time t are shown in the table above.

- (a) Use the data in the table to approximate the rate at which the temperature of the tea is changing at time $t = 3.5$. Show the computations that lead to your answer.
- (b) Using correct units, explain the meaning of $\frac{1}{10} \int_0^{10} H(t) dt$ in the context of this problem. Use a trapezoidal sum with the four subintervals indicated by the table to estimate $\frac{1}{10} \int_0^{10} H(t) dt$.
- (c) Evaluate $\int_0^{10} H'(t) dt$. Using correct units, explain the meaning of the expression in the context of this problem.
- (d) At time $t = 0$, biscuits with temperature 100°C were removed from an oven. The temperature of the biscuits at time t is modeled by a differentiable function B for which it is known that $B'(t) = -13.84e^{-0.173t}$. Using the given models, at time $t = 10$, how much cooler are the biscuits than the tea?

(a) $H'(3.5) \approx \frac{H(5) - H(2)}{5 - 2}$
 $= \frac{52 - 60}{3} = -2.666$ or -2.667 degrees Celsius per minute

1 : answer

(b) $\frac{1}{10} \int_0^{10} H(t) dt$ is the average temperature of the tea, in degrees Celsius, over the 10 minutes.

$$\frac{1}{10} \int_0^{10} H(t) dt \approx \frac{1}{10} \left(2 \cdot \frac{66 + 60}{2} + 3 \cdot \frac{60 + 52}{2} + 4 \cdot \frac{52 + 44}{2} + 1 \cdot \frac{44 + 43}{2} \right)$$

$$= 52.95$$

3 : $\begin{cases} 1 : \text{meaning of expression} \\ 1 : \text{trapezoidal sum} \\ 1 : \text{estimate} \end{cases}$

(c) $\int_0^{10} H'(t) dt = H(10) - H(0) = 43 - 66 = -23$
 The temperature of the tea drops 23 degrees Celsius from time $t = 0$ to time $t = 10$ minutes.

2 : $\begin{cases} 1 : \text{value of integral} \\ 1 : \text{meaning of expression} \end{cases}$

(d) $B(10) = 100 + \int_0^{10} B'(t) dt = 34.18275$; $H(10) - B(10) = 8.817$
 The biscuits are 8.817 degrees Celsius cooler than the tea.

3 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{uses } B(0) = 100 \\ 1 : \text{answer} \end{cases}$

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2011 SCORING GUIDELINES (Form B)

Question 1

A cylindrical can of radius 10 millimeters is used to measure rainfall in Stormville. The can is initially empty, and rain enters the can during a 60-day period. The height of water in the can is modeled by the function S , where $S(t)$ is measured in millimeters and t is measured in days for $0 \leq t \leq 60$. The rate at which the height of the water is rising in the can is given by $S'(t) = 2\sin(0.03t) + 1.5$.

- According to the model, what is the height of the water in the can at the end of the 60-day period?
- According to the model, what is the average rate of change in the height of water in the can over the 60-day period? Show the computations that lead to your answer. Indicate units of measure.
- Assuming no evaporation occurs, at what rate is the volume of water in the can changing at time $t = 7$? Indicate units of measure.
- During the same 60-day period, rain on Monsoon Mountain accumulates in a can identical to the one in Stormville. The height of the water in the can on Monsoon Mountain is modeled by the function M , where $M(t) = \frac{1}{400}(3t^3 - 30t^2 + 330t)$. The height $M(t)$ is measured in millimeters, and t is measured in days for $0 \leq t \leq 60$. Let $D(t) = M'(t) - S'(t)$. Apply the Intermediate Value Theorem to the function D on the interval $0 \leq t \leq 60$ to justify that there exists a time t , $0 < t < 60$, at which the heights of water in the two cans are changing at the same rate.

(a) $S(60) = \int_0^{60} S'(t) dt = 171.813 \text{ mm}$

3 : $\begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) $\frac{S(60) - S(0)}{60} = 2.863 \text{ or } 2.864 \text{ mm/day}$

1 : answer

(c) $V(t) = 100\pi S(t)$
 $V'(7) = 100\pi S'(7) = 602.218$

2 : $\begin{cases} 1 : \text{relationship between } V \text{ and } S \\ 1 : \text{answer} \end{cases}$

The volume of water in the can is increasing at a rate of $602.218 \text{ mm}^3/\text{day}$.

(d) $D(0) = -0.675 < 0$ and $D(60) = 69.37730 > 0$

2 : $\begin{cases} 1 : \text{considers } D(0) \text{ and } D(60) \\ 1 : \text{justification} \end{cases}$

Because D is continuous, the Intermediate Value Theorem implies that there is a time t , $0 < t < 60$, at which $D(t) = 0$. At this time, the heights of water in the two cans are changing at the same rate.

1 : units in (b) or (c)

AP[®] CALCULUS AB
2010 SCORING GUIDELINES

Question 1

There is no snow on Janet's driveway when snow begins to fall at midnight. From midnight to 9 A.M., snow accumulates on the driveway at a rate modeled by $f(t) = 7te^{\cos t}$ cubic feet per hour, where t is measured in hours since midnight. Janet starts removing snow at 6 A.M. ($t = 6$). The rate $g(t)$, in cubic feet per hour, at which Janet removes snow from the driveway at time t hours after midnight is modeled by

$$g(t) = \begin{cases} 0 & \text{for } 0 \leq t < 6 \\ 125 & \text{for } 6 \leq t < 7 \\ 108 & \text{for } 7 \leq t \leq 9. \end{cases}$$

- (a) How many cubic feet of snow have accumulated on the driveway by 6 A.M.?
 (b) Find the rate of change of the volume of snow on the driveway at 8 A.M.
 (c) Let $h(t)$ represent the total amount of snow, in cubic feet, that Janet has removed from the driveway at time t hours after midnight. Express h as a piecewise-defined function with domain $0 \leq t \leq 9$.
 (d) How many cubic feet of snow are on the driveway at 9 A.M.?

(a) $\int_0^6 f(t) dt = 142.274$ or 142.275 cubic feet

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Rate of change is $f(8) - g(8) = -59.582$ or -59.583 cubic feet per hour.

1 : answer

(c) $h(0) = 0$

For $0 < t \leq 6$, $h(t) = h(0) + \int_0^t g(s) ds = 0 + \int_0^t 0 ds = 0$.

For $6 < t \leq 7$, $h(t) = h(6) + \int_6^t g(s) ds = 0 + \int_6^t 125 ds = 125(t - 6)$.

For $7 < t \leq 9$, $h(t) = h(7) + \int_7^t g(s) ds = 125 + \int_7^t 108 ds = 125 + 108(t - 7)$.

Thus, $h(t) = \begin{cases} 0 & \text{for } 0 \leq t \leq 6 \\ 125(t - 6) & \text{for } 6 < t \leq 7 \\ 125 + 108(t - 7) & \text{for } 7 < t \leq 9 \end{cases}$

3 : $\begin{cases} 1 : h(t) \text{ for } 0 \leq t \leq 6 \\ 1 : h(t) \text{ for } 6 < t \leq 7 \\ 1 : h(t) \text{ for } 7 < t \leq 9 \end{cases}$

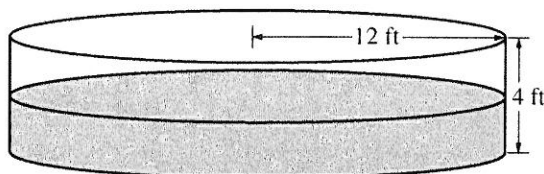
(d) Amount of snow is $\int_0^9 f(t) dt - h(9) = 26.334$ or 26.335 cubic feet.

3 : $\begin{cases} 1 : \text{integral} \\ 1 : h(9) \\ 1 : \text{answer} \end{cases}$

AP[®] CALCULUS AB
2010 SCORING GUIDELINES (Form B)

Question 3

t	0	2	4	6	8	10	12
$P(t)$	0	46	53	57	60	62	63



The figure above shows an aboveground swimming pool in the shape of a cylinder with a radius of 12 feet and a height of 4 feet. The pool contains 1000 cubic feet of water at time $t = 0$. During the time interval $0 \leq t \leq 12$ hours, water is pumped into the pool at the rate $P(t)$ cubic feet per hour. The table above gives values of $P(t)$ for selected values of t . During the same time interval, water is leaking from the pool at the rate $R(t)$ cubic feet per hour, where $R(t) = 25e^{-0.05t}$. (Note: The volume V of a cylinder with radius r and height h is given by $V = \pi r^2 h$.)

- Use a midpoint Riemann sum with three subintervals of equal length to approximate the total amount of water that was pumped into the pool during the time interval $0 \leq t \leq 12$ hours. Show the computations that lead to your answer.
- Calculate the total amount of water that leaked out of the pool during the time interval $0 \leq t \leq 12$ hours.
- Use the results from parts (a) and (b) to approximate the volume of water in the pool at time $t = 12$ hours. Round your answer to the nearest cubic foot.
- Find the rate at which the volume of water in the pool is increasing at time $t = 8$ hours. How fast is the water level in the pool rising at $t = 8$ hours? Indicate units of measure in both answers.

(a) $\int_0^{12} P(t) dt \approx 46 \cdot 4 + 57 \cdot 4 + 62 \cdot 4 = 660 \text{ ft}^3$

2 : $\begin{cases} 1 : \text{midpoint sum} \\ 1 : \text{answer} \end{cases}$

(b) $\int_0^{12} R(t) dt = 225.594 \text{ ft}^3$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(c) $1000 + \int_0^{12} P(t) dt - \int_0^{12} R(t) dt = 1434.406$

1 : answer

At time $t = 12$ hours, the volume of water in the pool is approximately 1434 ft^3 .

(d) $V'(t) = P(t) - R(t)$
 $V'(8) = P(8) - R(8) = 60 - 25e^{-0.4} = 43.241$ or $43.242 \text{ ft}^3/\text{hr}$

$V = \pi(12)^2 h$

$\frac{dV}{dt} = 144\pi \frac{dh}{dt}$

$\left. \frac{dh}{dt} \right|_{t=8} = \frac{1}{144\pi} \cdot \left. \frac{dV}{dt} \right|_{t=8} = 0.095$ or 0.096 ft/hr

4 : $\begin{cases} 1 : V'(8) \\ 1 : \text{equation relating } \frac{dV}{dt} \text{ and } \frac{dh}{dt} \\ 1 : \left. \frac{dh}{dt} \right|_{t=8} \\ 1 : \text{units of } \text{ft}^3/\text{hr} \text{ and } \text{ft/hr} \end{cases}$

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2009 SCORING GUIDELINES

Question 2

The rate at which people enter an auditorium for a rock concert is modeled by the function R given by $R(t) = 1380t^2 - 675t^3$ for $0 \leq t \leq 2$ hours; $R(t)$ is measured in people per hour. No one is in the auditorium at time $t = 0$, when the doors open. The doors close and the concert begins at time $t = 2$.

- How many people are in the auditorium when the concert begins?
- Find the time when the rate at which people enter the auditorium is a maximum. Justify your answer.
- The total wait time for all the people in the auditorium is found by adding the time each person waits, starting at the time the person enters the auditorium and ending when the concert begins. The function w models the total wait time for all the people who enter the auditorium before time t . The derivative of w is given by $w'(t) = (2 - t)R(t)$. Find $w(2) - w(1)$, the total wait time for those who enter the auditorium after time $t = 1$.
- On average, how long does a person wait in the auditorium for the concert to begin? Consider all people who enter the auditorium after the doors open, and use the model for total wait time from part (c).

(a) $\int_0^2 R(t) dt = 980$ people

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

- (b) $R'(t) = 0$ when $t = 0$ and $t = 1.36296$
 The maximum rate may occur at 0, $a = 1.36296$, or 2.

$$\begin{aligned} R(0) &= 0 \\ R(a) &= 854.527 \\ R(2) &= 120 \end{aligned}$$

The maximum rate occurs when $t = 1.362$ or 1.363 .

3 : $\begin{cases} 1 : \text{considers } R'(t) = 0 \\ 1 : \text{interior critical point} \\ 1 : \text{answer and justification} \end{cases}$

(c) $w(2) - w(1) = \int_1^2 w'(t) dt = \int_1^2 (2 - t)R(t) dt = 387.5$

The total wait time for those who enter the auditorium after time $t = 1$ is 387.5 hours.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) $\frac{1}{980} w(2) = \frac{1}{980} \int_0^2 (2 - t)R(t) dt = 0.77551$

On average, a person waits 0.775 or 0.776 hour.

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

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2009 SCORING GUIDELINES (Form B)

Question 2

A storm washed away sand from a beach, causing the edge of the water to get closer to a nearby road. The rate at which the distance between the road and the edge of the water was changing during the storm is modeled by $f(t) = \sqrt{t} + \cos t - 3$ meters per hour, t hours after the storm began. The edge of the water was 35 meters from the road when the storm began, and the storm lasted 5 hours. The derivative of $f(t)$ is $f'(t) = \frac{1}{2\sqrt{t}} - \sin t$.

- What was the distance between the road and the edge of the water at the end of the storm?
- Using correct units, interpret the value $f'(4) = 1.007$ in terms of the distance between the road and the edge of the water.
- At what time during the 5 hours of the storm was the distance between the road and the edge of the water decreasing most rapidly? Justify your answer.
- After the storm, a machine pumped sand back onto the beach so that the distance between the road and the edge of the water was growing at a rate of $g(p)$ meters per day, where p is the number of days since pumping began. Write an equation involving an integral expression whose solution would give the number of days that sand must be pumped to restore the original distance between the road and the edge of the water.

(a) $35 + \int_0^5 f(t) dt = 26.494$ or 26.495 meters

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

- (b) Four hours after the storm began, the rate of change of the distance between the road and the edge of the water is increasing at a rate of 1.007 meters/hours².

2 : $\begin{cases} 1 : \text{interpretation of } f'(4) \\ 1 : \text{units} \end{cases}$

- (c) $f'(t) = 0$ when $t = 0.66187$ and $t = 2.84038$
 The minimum of f for $0 \leq t \leq 5$ may occur at 0, 0.66187, 2.84038, or 5.

3 : $\begin{cases} 1 : \text{considers } f'(t) = 0 \\ 1 : \text{answer} \\ 1 : \text{justification} \end{cases}$

$$f(0) = -2$$

$$f(0.66187) = -1.39760$$

$$f(2.84038) = -2.26963$$

$$f(5) = -0.48027$$

The distance between the road and the edge of the water was decreasing most rapidly at time $t = 2.840$ hours after the storm began.

(d) $-\int_0^5 f(t) dt = \int_0^x g(p) dp$

2 : $\begin{cases} 1 : \text{integral of } g \\ 1 : \text{answer} \end{cases}$

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2008 SCORING GUIDELINES

Question 2

t (hours)	0	1	3	4	7	8	9
$L(t)$ (people)	120	156	176	126	150	80	0

Concert tickets went on sale at noon ($t = 0$) and were sold out within 9 hours. The number of people waiting in line to purchase tickets at time t is modeled by a twice-differentiable function L for $0 \leq t \leq 9$. Values of $L(t)$ at various times t are shown in the table above.

- (a) Use the data in the table to estimate the rate at which the number of people waiting in line was changing at 5:30 P.M. ($t = 5.5$). Show the computations that lead to your answer. Indicate units of measure.
- (b) Use a trapezoidal sum with three subintervals to estimate the average number of people waiting in line during the first 4 hours that tickets were on sale.
- (c) For $0 \leq t \leq 9$, what is the fewest number of times at which $L'(t)$ must equal 0? Give a reason for your answer.
- (d) The rate at which tickets were sold for $0 \leq t \leq 9$ is modeled by $r(t) = 550te^{-t/2}$ tickets per hour. Based on the model, how many tickets were sold by 3 P.M. ($t = 3$), to the nearest whole number?

(a) $L'(5.5) \approx \frac{L(7) - L(4)}{7 - 4} = \frac{150 - 126}{3} = 8$ people per hour

2 : $\begin{cases} 1 : \text{estimate} \\ 1 : \text{units} \end{cases}$

- (b) The average number of people waiting in line during the first 4 hours is approximately

$$\frac{1}{4} \left(\frac{L(0) + L(1)}{2}(1 - 0) + \frac{L(1) + L(3)}{2}(3 - 1) + \frac{L(3) + L(4)}{2}(4 - 3) \right) = 155.25 \text{ people}$$

2 : $\begin{cases} 1 : \text{trapezoidal sum} \\ 1 : \text{answer} \end{cases}$

- (c) L is differentiable on $[0, 9]$ so the Mean Value Theorem implies $L'(t) > 0$ for some t in $(1, 3)$ and some t in $(4, 7)$. Similarly, $L'(t) < 0$ for some t in $(3, 4)$ and some t in $(7, 8)$. Then, since L' is continuous on $[0, 9]$, the Intermediate Value Theorem implies that $L'(t) = 0$ for at least three values of t in $[0, 9]$.

3 : $\begin{cases} 1 : \text{considers change in sign of } L' \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$

OR

The continuity of L on $[1, 4]$ implies that L attains a maximum value there. Since $L(3) > L(1)$ and $L(3) > L(4)$, this maximum occurs on $(1, 4)$. Similarly, L attains a minimum on $(3, 7)$ and a maximum on $(4, 8)$. L is differentiable, so $L'(t) = 0$ at each relative extreme point on $(0, 9)$. Therefore $L'(t) = 0$ for at least three values of t in $[0, 9]$.

OR

3 : $\begin{cases} 1 : \text{considers relative extrema of } L \text{ on } (0, 9) \\ 1 : \text{analysis} \\ 1 : \text{conclusion} \end{cases}$

[Note: There is a function L that satisfies the given conditions with $L'(t) = 0$ for exactly three values of t .]

(d) $\int_0^3 r(t) dt = 972.784$

There were approximately 973 tickets sold by 3 P.M.

2 : $\begin{cases} 1 : \text{integrand} \\ 1 : \text{limits and answer} \end{cases}$

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Question 3

Distance from the river's edge (feet)	0	8	14	22	24
Depth of the water (feet)	0	7	8	2	0

A scientist measures the depth of the Doe River at Picnic Point. The river is 24 feet wide at this location. The measurements are taken in a straight line perpendicular to the edge of the river. The data are shown in the table above. The velocity of the water at Picnic Point, in feet per minute, is modeled by

$$v(t) = 16 + 2\sin(\sqrt{t+10}) \text{ for } 0 \leq t \leq 120 \text{ minutes.}$$

- (a) Use a trapezoidal sum with the four subintervals indicated by the data in the table to approximate the area of the cross section of the river at Picnic Point, in square feet. Show the computations that lead to your answer.
- (b) The volumetric flow at a location along the river is the product of the cross-sectional area and the velocity of the water at that location. Use your approximation from part (a) to estimate the average value of the volumetric flow at Picnic Point, in cubic feet per minute, from $t = 0$ to $t = 120$ minutes.
- (c) The scientist proposes the function f , given by $f(x) = 8\sin\left(\frac{\pi x}{24}\right)$, as a model for the depth of the water, in feet, at Picnic Point x feet from the river's edge. Find the area of the cross section of the river at Picnic Point based on this model.
- (d) Recall that the volumetric flow is the product of the cross-sectional area and the velocity of the water at a location. To prevent flooding, water must be diverted if the average value of the volumetric flow at Picnic Point exceeds 2100 cubic feet per minute for a 20-minute period. Using your answer from part (c), find the average value of the volumetric flow during the time interval $40 \leq t \leq 60$ minutes. Does this value indicate that the water must be diverted?

$$(a) \frac{(0+7)}{2} \cdot 8 + \frac{(7+8)}{2} \cdot 6 + \frac{(8+2)}{2} \cdot 8 + \frac{(2+0)}{2} \cdot 2$$

$$= 115 \text{ ft}^2$$

1 : trapezoidal approximation

$$(b) \frac{1}{120} \int_0^{120} 115v(t) dt$$

$$= 1807.169 \text{ or } 1807.170 \text{ ft}^3/\text{min}$$

3 : $\begin{cases} 1 : \text{limits and average value} \\ \text{constant} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

$$(c) \int_0^{24} 8\sin\left(\frac{\pi x}{24}\right) dx = 122.230 \text{ or } 122.231 \text{ ft}^2$$

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(d) Let C be the cross-sectional area approximation from part (c). The average volumetric flow is

$$\frac{1}{20} \int_{40}^{60} C \cdot v(t) dt = 2181.912 \text{ or } 2181.913 \text{ ft}^3/\text{min.}$$

3 : $\begin{cases} 1 : \text{volumetric flow integral} \\ 1 : \text{average volumetric flow} \\ 1 : \text{answer with reason} \end{cases}$

Yes, water must be diverted since the average volumetric flow for this 20-minute period exceeds $2100 \text{ ft}^3/\text{min}$.

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Question 2

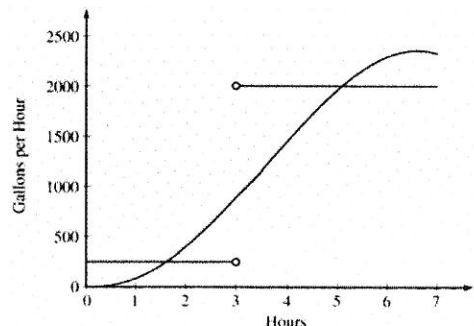
The amount of water in a storage tank, in gallons, is modeled by a continuous function on the time interval $0 \leq t \leq 7$, where t is measured in hours. In this model, rates are given as follows:

(i) The rate at which water enters the tank is

$$f(t) = 100t^2 \sin(\sqrt{t}) \text{ gallons per hour for } 0 \leq t \leq 7.$$

(ii) The rate at which water leaves the tank is

$$g(t) = \begin{cases} 250 & \text{for } 0 \leq t < 3 \\ 2000 & \text{for } 3 < t \leq 7 \end{cases} \text{ gallons per hour.}$$



The graphs of f and g , which intersect at $t = 1.617$ and $t = 5.076$, are shown in the figure above. At time $t = 0$, the amount of water in the tank is 5000 gallons.

- (a) How many gallons of water enter the tank during the time interval $0 \leq t \leq 7$? Round your answer to the nearest gallon.
- (b) For $0 \leq t \leq 7$, find the time intervals during which the amount of water in the tank is decreasing. Give a reason for each answer.
- (c) For $0 \leq t \leq 7$, at what time t is the amount of water in the tank greatest? To the nearest gallon, compute the amount of water at this time. Justify your answer.

(a) $\int_0^7 f(t) dt \approx 8264$ gallons

2 : $\begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

- (b) The amount of water in the tank is decreasing on the intervals $0 \leq t \leq 1.617$ and $3 \leq t \leq 5.076$ because $f(t) < g(t)$ for $0 \leq t < 1.617$ and $3 < t < 5.076$.

2 : $\begin{cases} 1 : \text{intervals} \\ 1 : \text{reason} \end{cases}$

- (c) Since $f(t) - g(t)$ changes sign from positive to negative only at $t = 3$, the candidates for the absolute maximum are at $t = 0$, 3, and 7.

5 : $\begin{cases} 1 : \text{identifies } t = 3 \text{ as a candidate} \\ 1 : \text{integrand} \\ 1 : \text{amount of water at } t = 3 \\ 1 : \text{amount of water at } t = 7 \\ 1 : \text{conclusion} \end{cases}$

t (hours)	gallons of water
0	5000
3	$5000 + \int_0^3 f(t) dt - 250(3) = 5126.591$
7	$5126.591 + \int_3^7 f(t) dt - 2000(4) = 4513.807$

The amount of water in the tank is greatest at 3 hours. At that time, the amount of water in the tank, rounded to the nearest gallon, is 5127 gallons.

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Question 3

The wind chill is the temperature, in degrees Fahrenheit ($^{\circ}\text{F}$), a human feels based on the air temperature, in degrees Fahrenheit, and the wind velocity v , in miles per hour (mph). If the air temperature is 32°F , then the wind chill is given by $W(v) = 55.6 - 22.1v^{0.16}$ and is valid for $5 \leq v \leq 60$.

- (a) Find $W'(20)$. Using correct units, explain the meaning of $W'(20)$ in terms of the wind chill.
- (b) Find the average rate of change of W over the interval $5 \leq v \leq 60$. Find the value of v at which the instantaneous rate of change of W is equal to the average rate of change of W over the interval $5 \leq v \leq 60$.
- (c) Over the time interval $0 \leq t \leq 4$ hours, the air temperature is a constant 32°F . At time $t = 0$, the wind velocity is $v = 20$ mph. If the wind velocity increases at a constant rate of 5 mph per hour, what is the rate of change of the wind chill with respect to time at $t = 3$ hours? Indicate units of measure.

(a) $W'(20) = -22.1 \cdot 0.16 \cdot 20^{-0.84} = -0.285$ or -0.286

When $v = 20$ mph, the wind chill is decreasing at $0.286^{\circ}\text{F}/\text{mph}$.

2 : $\begin{cases} 1 : \text{value} \\ 1 : \text{explanation} \end{cases}$

(b) The average rate of change of W over the interval $5 \leq v \leq 60$ is $\frac{W(60) - W(5)}{60 - 5} = -0.253$ or -0.254 .

$W'(v) = \frac{W(60) - W(5)}{60 - 5}$ when $v = 23.011$.

3 : $\begin{cases} 1 : \text{average rate of change} \\ 1 : W'(v) = \text{average rate of change} \\ 1 : \text{value of } v \end{cases}$

(c) $\left. \frac{dW}{dt} \right|_{t=3} = \left(\frac{dW}{dv} \cdot \frac{dv}{dt} \right) \Big|_{t=3} = W'(35) \cdot 5 = -0.892^{\circ}\text{F}/\text{hr}$

OR

$W = 55.6 - 22.1(20 + 5t)^{0.16}$

$\left. \frac{dW}{dt} \right|_{t=3} = -0.892^{\circ}\text{F}/\text{hr}$

3 : $\begin{cases} 1 : \frac{dv}{dt} = 5 \\ 1 : \text{uses } v(3) = 35, \\ \text{or} \\ \text{uses } v(t) = 20 + 5t \\ 1 : \text{answer} \end{cases}$

Units of $^{\circ}\text{F}/\text{mph}$ in (a) and $^{\circ}\text{F}/\text{hr}$ in (c)

1 : units in (a) and (c)